Influence of the geometry of the feature space on curiosity based exploration

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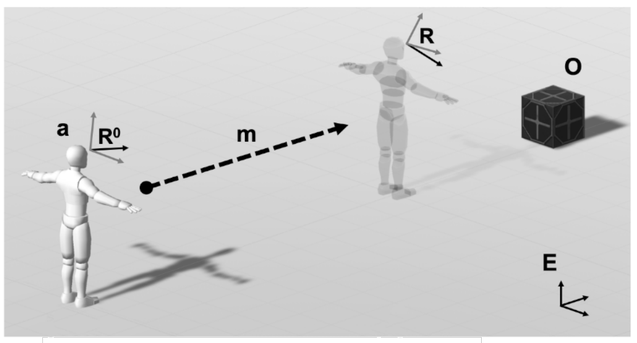
Intro : In human spatial awareness ...

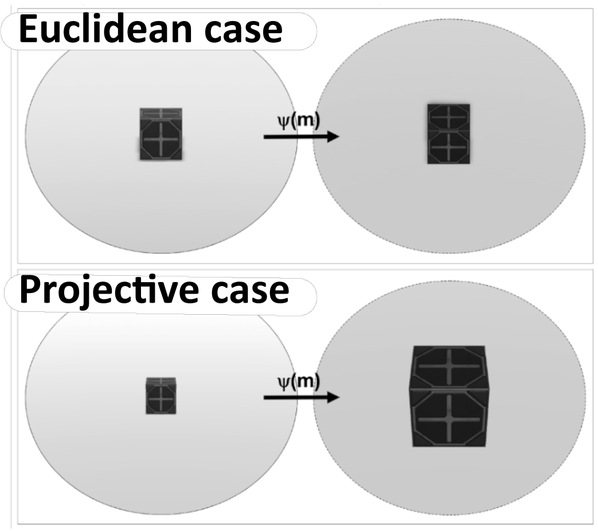
- Information is represented according to **3D Projective geometry** → structures **information integration**/action **planning**
- **Subjective perspective** can be taken on this **internal representation space** = projective transformations

⇒ How do **different geometries** affect **exploration behavior ?**

Methods

- **Agent** a looks for an **object** O
- **Euclidean vs Projective** geometry as internal reprensentation spaces





Group Structured World Model

- Information integration → **internal world model** W
- **Changes of perspective** are formalized by **group actions**
- ψ_m : maps world model to the internal space

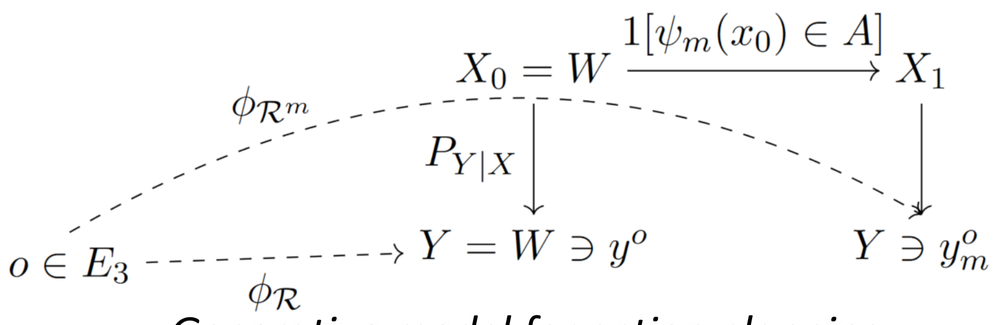
W is a group structured world model for the group G when there is a map $h : G \times W \to W \quad h(g, x) = g.x$ for $g \in G$ and $x \in X$

1. $(g.g_1).x = g.(g_1.x)$ *for all* $g, g_1 \in G,\ x \in W$
2. $e.x = x$, *for all* $x \in W$

Beliefs and observations

- **Beliefs** about the position of O = **probability measure** on W
- Beliefs are updated according to **noisy observations** of O
- Uncertainty of sensors = captured by Markov Kernel $P_{Y|X}$ (Y = observation space)
- with beliefs $Q_X \in \mathbb{P}(W)$, observation $y^o \in W$, the **updated beliefs** are :

$$Q_{X|y^o} = \frac{p_{Y|X}(y^o|x)q_X(x)dx}{\int_{x\in W} p_{Y|X}(y^o|x)q_X(x)dx}$$



Generative model for action planning

Policies

- Agent a has a **set of moves** M (eg. translations)
- $m \in M$ = **group element**, $\psi_m : W \to W$ (change of perspective)
- **Plans** the consequence **one step ahead** = the beliefs Q_X are transformed to account for the **new perspective** of a move m

for any $m \in M$, $A \subseteq W$ a (Borel) subset of W, and $x_0 \in W$,

$p_{X_1|X_0,m}(A|x_0, m) = 1[\psi_m(x_0) \in A]$

$$\psi_{m,*}Q_X(A) := \int 1[\psi_m(x_0) \in A]q_X(x_0)dx = Q_X(\psi_m^{-1}A)$$

Exploration algorithm

- **Curiosity based** exploration : selecting the move maximizing the epistemic value
- **Epistemic value** = **information gain** = relative entropy of a belief distribution (Kullback-Leibler Divergence)

$$C(Q_X) := \mathbb{E}_{P_Y}\left[H(P_{X|Y}|Q_X)\right] = \int p_{X,Y} \ln \frac{p_{X,Y}(x,y)}{p_Y(y)q_X(x)} dxdy$$

- **Value of a move** = value of the **induced beliefs**
- Exploration algorithm : select a **move** → compute **induced beliefs** → receive an **observation** and **update** the beliefs

Algorithm 1: Curiosity based Exploration for agent a

Data: Initialization: Q_X^0 initial belief, $\mathcal{R}^0$ initial solid reference frame of a

$Q_X \leftarrow Q_X^0$;
while *True* **do**
  $\overline{m} \leftarrow \text{argmax}_{m\in M} C(\psi_{m,*}Q_X)$;
  $\mathcal{R} \leftarrow$ solid reference frame of a after move $\overline{m}$;
  $Q_X \leftarrow \psi_{\overline{m},*}Q_X$;
  $y^o \leftarrow \phi_{\mathcal{R}}(o)$;
  $Q_X \leftarrow Q_{X|y^o}$;
end

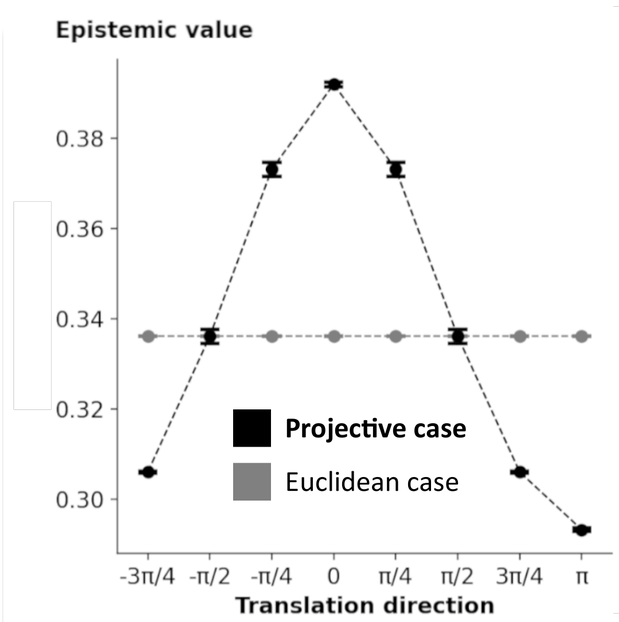
Theoretical predictions and results

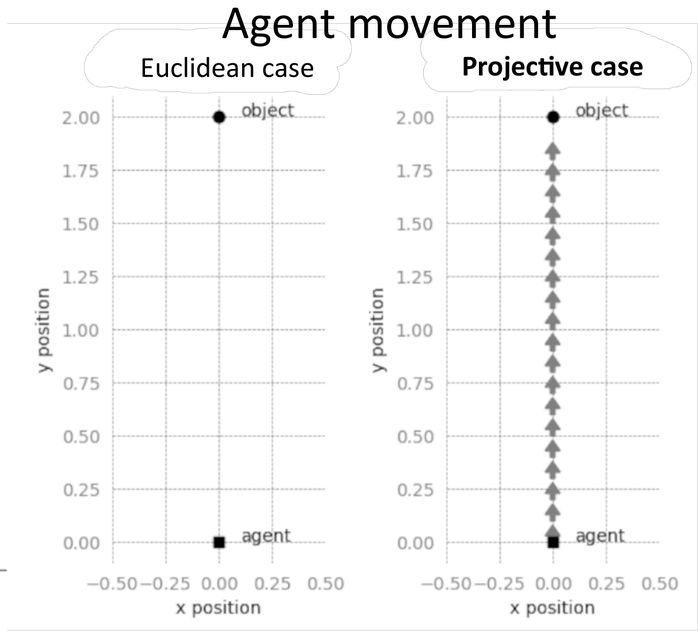
- **Theoretical prediction**

Euclidean case → agent **stays still**
Projective case → agent **approaches the object**

- **Intuition** (proof in the paper)

Euclidean case : the object always **appears** to be the **same size** → all perspectives hold the **same information**
Projective case : **approaching** the region of the object "**zooms**" on it → **observation** are more **accurate** = more **information**





Epistemic value and exploration behavior from simulations

Conclusion

- We proposed a **generative model** based on **subjective perspective** structured by a **group** (geometry), with actions = changes of perspective
- **Different groups** = **different behaviors**
- **Future work** :

→ Deep learning for group structured features
→ Reproducing other features of consciousness

Rudrauf et al., The Projective Consciousness Model: Projective Geometry at the Core of Consciousness and the Integration of Perception, Imagination, Motivation, Emotion, Social Cognition and Action, 2023
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